## **Soft Actor-Critic**



Zikun Chen, Minghan Li Jan. 28, 2020

## Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor

Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, Sergey Levine

## Outline

- Problem: Sample Efficiency
- Solution: Off-Policy Learning
  - On-Policy vs Off-Policy
  - RL Basics Recap
  - Off-Policy Learning Algorithms
- Problem: Robustness
- Solution: Maximum Entropy RL
  - Definition (Control as Inference)
  - Soft Policy Iteration
  - Soft Actor-Critic

#### Contributions

- An off-policy maximum entropy deep reinforcement learning algorithm
  - Sample-efficient
  - Robustness to noise, random seed and hyperparameters
  - Scale to high-dimensional observation/action space
- Theoretical Results
  - Theoretical framework of soft policy iteration
  - Derivation of soft-actor critic algorithm
- Empirical Results
  - SAC outperforms SOTA model-free deep RL methods, including DDPG, PPO and Soft Q-learning, in terms of the policy's optimality, sample complexity and stability.

## Outline

- Problem: Sample Efficiency
- Solution: Off-Policy Learning
  - On-Policy vs Off-Policy
  - RL Basics Recap
  - Off-Policy Learning Algorithms

- Number of times the agent must interact with the environment in order to learn a task
- Good sample complexity is the first prerequisite for successful skill acquisition.
- Learning skills in the real world can take a substantial amount of time
  - can get damaged through trial and error

- "Learning Hand-Eye Coordination for Robotic Grasping with Deep Learning and Large-Scale Data Collection", Levine et al., 2016
  - 14 robot arms learning to grasp in parallel
  - o objects started being picked up at around 20,000 grasps



https://spectrum.ieee.org/automaton/robotics/ artificial-intelligence/google-large-scale-roboti c-grasping-project

Observing the behavior of the robot after over 800,000 grasp attempts, which is equivalent to about 3000 robothours of practice, we can see the beginnings of intelligent reactive behaviors



- Solution?
- Off-Policy Learning!

## Background: On-Policy vs. Off-Policy

- On-policy learning: use the deterministic outcomes or samples from the target policy to train the algorithm
  - has low sample efficiency (TRPO, PPO, A3C)
  - require new samples to be collected for nearly every update to the policy
  - becomes extremely expensive when the task is complex
- Off-policy methods: training on a distribution of transitions or episodes produced by a different behavior policy rather than that produced by the target policy
  - Does not require full trajectories and can reuse any past episodes (experience replay) for much better sample efficiency
  - relatively straightforward for Q-learning based methods

#### Background: Bellman Equation

• Value Function: How good is a state?

$$V(s) = \mathbb{E}[G_t | S_t = s]$$
  
=  $\mathbb{E}[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots | S_t = s]$   
=  $\mathbb{E}[R_{t+1} + \gamma (R_{t+2} + \gamma R_{t+3} + \dots) | S_t = s]$   
=  $\mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s]$   
=  $\mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s]$ 

temporal difference target

• Similarly, for Q-Function: How good is a state-action pair?

$$Q(s, a) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) \mid S_t = s, A_t = a] \\= \mathbb{E}[R_{t+1} + \gamma \mathbb{E}_{a \sim \pi} Q(S_{t+1}, a) \mid S_t = s, A_t = a]$$

#### Background: Value-Based Method

.

• ..., 
$$S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, ...$$
 (on-policy):  
 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$ 

- Q-Learning (off-policy)  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a \in \mathcal{A}} Q(S_{t+1}, a) - Q(S_t, A_t))$
- DQN, Minh et al., 2015

$$\mathcal{L}(\theta) = \mathbb{E}_{(s,a,r,s') \sim U(D)} \left[ \left( r + \gamma \max_{a'} Q(s',a';\theta^{-}) - Q(s,a;\theta) \right)^2 \right]$$

- Function Approximation
- Experience Replay: samples randomly drawn from replay memory
- Doesn't scale to continuous action space



#### Background: Policy-Based Method (Actor-Critic)

- **Critic**: updates value function parameters w and depending on the algorithm it could be action-value Q(a|s; w) or state-value V(s; w).
- Actor: updates policy parameters  $\theta$ , in the direction suggested by the critic,  $\pi(a|s; \theta)$ .

$$\nabla \mathcal{J}(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla \ln \pi(a|s,\theta) Q_{\pi}(s,a)] \quad \text{policy gradient}$$

$$\theta \leftarrow \theta + \alpha_{\theta} Q(s, a; w) \nabla_{\theta} \ln \pi(a|s; \theta)$$
 update actor

 $G_{t:t+1} = r_t + \gamma Q(s', a'; w) - Q(s, a; w)$  correction for action-value

 $w \leftarrow w + \alpha_w G_{t:t+1} \nabla_w Q(s, a; w)$ . update critic

## Prior Work: DDPG

- DDPG = DQN + DPG (Lillicrap et al., 2015)
  - off-policy actor-critic method that learns a deterministic policy in <u>continuous</u> domain
  - exploration noise added to the deterministic policy when select action
  - difficult to stabilize and brittle to hyperparameters (Duan et al., 2016, Henderson et al., 2017)
  - unscalable to complex tasks with high dimensions (Gu et al., 2017)



#### Outline

- Problem: Sample Efficiency
- Solution: Off-Policy Learning
  - On-Policy vs Off-Policy
  - RL Basics Recap
  - Off-Policy Learning Algorithms

#### Problem: Robustness

- Solution: Maximum Entropy RL
  - Definition (Control as Inference)
  - Soft Policy Iteration
  - Soft Actor-Critic

#### Main Problems: Robustness

• Training is sensitive to randomness in the environment, initialization of the policy and the algorithm implementation



#### Main Problems: Robustness

• Knowing only one way to act makes agents vulnerable to environmental changes that are common in the real-world



#### **Background: Control as Inference**





Traditional Graph of MDP

Graphical Model with Optimality Variables

#### **Background: Control as Inference**

Normal trajectory distribution

$$p(\tau) = p(\mathbf{s}_1, \mathbf{a}_t, \dots, \mathbf{s}_T, \mathbf{a}_T | \theta) = p(\mathbf{s}_1) \prod_{t=1}^T p(\mathbf{a}_t | \mathbf{s}_t, \theta) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t).$$

Posterior trajectory distribution

$$p(\mathcal{O}_t = 1 | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t)).$$

$$p(\tau | \mathbf{o}_{1:T}) \propto p(\tau, \mathbf{o}_{1:T}) = p(\mathbf{s}_1) \prod_{t=1}^T p(\mathcal{O}_t = 1 | \mathbf{s}_t, \mathbf{a}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$
$$= p(\mathbf{s}_1) \prod_{t=1}^T \exp(r(\mathbf{s}_t, \mathbf{a}_t)) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$
$$= \left[ p(\mathbf{s}_1) \prod_{t=1}^T p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t) \right] \exp\left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t)\right)$$

#### Background: Control as Inference

Variational Inference

$$D_{\mathrm{KL}}(\hat{p}(\tau)||p(\tau)) = -E_{\tau \sim \hat{p}(\tau)} [\log p(\tau) - \log \hat{p}(\tau)].$$
  
$$-D_{\mathrm{KL}}(\hat{p}(\tau)||p(\tau)) = E_{\tau \sim \hat{p}(\tau)} \left[ \log p(\mathbf{s}_{1}) + \sum_{t=1}^{T} (\log p(\mathbf{s}_{t+1}|\mathbf{s}_{t}, \mathbf{a}_{t}) + r(\mathbf{s}_{t}, \mathbf{a}_{t})) - \log p(\mathbf{s}_{1}) - \sum_{t=1}^{T} (\log p(\mathbf{s}_{t+1}|\mathbf{s}_{t}, \mathbf{a}_{t}) + \log \pi(\mathbf{a}_{t}|\mathbf{s}_{t})) \right]$$
  
$$= E_{\tau \sim \hat{p}(\tau)} \left[ \sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) - \log \pi(\mathbf{a}_{t}|\mathbf{s}_{t}) \right]$$
  
$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim \hat{p}(\mathbf{s}_{t}, \mathbf{a}_{t}))} [r(\mathbf{s}_{t}, \mathbf{a}_{t}) - \log \pi(\mathbf{a}_{t}|\mathbf{s}_{t})]$$
  
$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim \hat{p}(\mathbf{s}_{t}, \mathbf{a}_{t}))} [r(\mathbf{s}_{t}, \mathbf{a}_{t})] + E_{\mathbf{s}_{t} \sim \hat{p}(\mathbf{s}_{t})} [\mathcal{H}(\pi(\mathbf{a}_{t}|\mathbf{s}_{t}))].$$

#### Background: Max Entropy RL

Conventional RL Objective - Expected Reward

$$\sum_{t} \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \rho_{\pi}} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

Maximum Entropy RL Objective - Expected Reward + Entropy of Policy

$$\sum_{t} \mathbb{E}_{(\mathbf{s}_{t},\mathbf{a}_{t})\sim\rho_{\pi}} \left[ r(\mathbf{s}_{t},\mathbf{a}_{t}) + \alpha \mathcal{H}(\pi(\cdot|\mathbf{s}_{t})) \right]$$

Entropy of a RV x

$$H(P) = \mathop{\mathrm{E}}_{x \sim P} \left[ -\log P(x) \right]'$$

## Max Entropy RL

• MaxEnt RL agent can capture different modes of optimality to improve robustness against environmental changes









https://bair.berkeley.edu/blog/2017/10/06/soft-q-learning/

#### Max Entropy RL



$$\begin{split} \min_{\pi} D_{KL}[\pi(\cdot|s_0)||\exp(Q(s_0,\cdot))] \\ &= \min_{\pi} E_{\pi}[\log \frac{\pi(a_0|s_0)}{\exp(Q(s_0,a_0))}] \\ &= \max_{\pi} E_{\pi}[Q(s_0,a_0) - \log \pi(a_0|s_0)] \\ &= \max_{\pi} E_{\pi}[\sum_{t} r(s_t,a_t) + \mathcal{H}(\pi(\cdot|s_0))|s_0] \\ &= J_{MaxEnt}(\pi(\cdot|s_0)) \end{split}$$

#### Prior Work: Soft Q-Learning

- Soft Q-Learning (Haarnoja et al., 2017)
  - off-policy algorithms under MaxEnt RL objective
  - Learns Q\* directly
  - $\circ$  sample policy from exp(Q\*) is intractable for continuous actions
  - use approximate inference methods to sample
    - Stein variational gradient descent
  - not true actor-critic

 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a \in \mathcal{A}} Q(S_{t+1}, a) - Q(S_t, A_t))$ 

$$Q_{\text{soft}}(\mathbf{s}_{t}, \mathbf{a}_{t}) \leftarrow r_{t} + \gamma \mathbb{E}_{\mathbf{s}_{t+1} \sim p_{\mathbf{s}}} \left[ V_{\text{soft}}(\mathbf{s}_{t+1}) \right], \ \forall \mathbf{s}_{t}, \mathbf{a}_{t}$$
$$V_{\text{soft}}(\mathbf{s}_{t}) \leftarrow \alpha \log \int_{\mathcal{A}} \exp \left( \frac{1}{\alpha} Q_{\text{soft}}(\mathbf{s}_{t}, \mathbf{a}') \right) d\mathbf{a}', \ \forall \mathbf{s}_{t}$$

#### **SAC:** Contributions

- One of the most efficient model-free algorithms
  - SOTA off-policy
  - well suited for real world robotics learning
- Can learn stochastic policy on continuous action domain
- Robust to noise
- Ingredients:
  - Actor-critic architecture with seperate policy and value function networks
  - Off-policy formulation to reuse of previously collected data for efficiency
  - Entropy-constrained objective to encourage stability and exploration

#### Soft Policy Iteration: Policy Evaluation

- policy evaluation: compute value of π according to Max Entropy RL Objective
- modified Bellman backup operator T:

$$\mathcal{T}^{\pi}Q(\mathbf{s}_{t}, \mathbf{a}_{t}) \triangleq r(\mathbf{s}_{t}, \mathbf{a}_{t}) + \gamma \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[ V(\mathbf{s}_{t+1}) \right]$$
$$V(\mathbf{s}_{t}) = \mathbb{E}_{\mathbf{a}_{t} \sim \pi} \left[ Q(\mathbf{s}_{t}, \mathbf{a}_{t}) - \alpha \log \pi(\mathbf{a}_{t} | \mathbf{s}_{t}) \right]$$

• Lemma 1: Contraction Mapping for Soft Bellman Updates

$$Q^{k+1} = \mathcal{T}^{\pi}Q^k$$
 , converges to the soft Q-function of T

#### Soft Policy Iteration: Policy Improvement

- policy improvement: update policy towards the exponential of the new soft Q-function
- modified Bellman backup operator T:
  - $\circ$  choose tractable family of distributions big  $\Pi$
  - $\circ$  choose KL divergence to project the improved policy into big  $\Pi$

$$\pi_{\text{new}} = \arg\min_{\pi' \in \Pi} D_{\text{KL}} \left( \pi'(\cdot | \mathbf{s}_t) \left\| \frac{\exp\left(\frac{1}{\alpha} Q^{\pi_{\text{old}}}(\mathbf{s}_t, \cdot)\right)}{Z^{\pi_{\text{old}}}(\mathbf{s}_t)} \right) \right) \xrightarrow{\mathcal{I}_{Q(\mathbf{s}_t, \mathbf{a}_t)}}{\mathbf{s}_t} \left( \frac{\mathcal{I}_{Q(\mathbf{s}_t, \mathbf{a}_t)}}{\mathcal{I}_{Q(\mathbf{s}_t, \mathbf{a}_t)}} \right)$$

• Lemma 2

$$Q^{\pi_{ ext{new}}}(\mathbf{s}_t,\mathbf{a}_t) \geq Q^{\pi_{ ext{old}}}(\mathbf{s}_t,\mathbf{a}_t)$$
 for any state action pair

#### **Soft Policy Iteration**

- soft policy iteration: soft policy evaluation <-> soft policy improvement
- Theorem 1: Repeated application of soft policy evaluation and soft policy improvement from any policy  $\pi \in \Pi$  converges to the optimal MaxEnt policy among all policies in  $\Pi$ 
  - $\circ$  exact form applicable only in discrete case
  - need function approximation to represent Q-values in continuous domains
  - -> Soft Actor-Critic (SAC)!

SAC

 $Q_{\theta}(\mathbf{s}_t, \mathbf{a}_t)$  $\pi_{\phi}(\mathbf{a}_t|\mathbf{s}_t)$  $\hat{\nabla}_{\theta} J_Q(\theta)$  $J_O(\theta)$  $J_{\pi}(\phi) = \hat{\nabla}_{\phi} J_{\pi}(\phi)$ 

parameterized soft Q-function

• e.g.neural network

parameterized tractable policy

• e.g. Gaussian with mean and covariances given by neural networks

soft Q-function objective and its stochastic gradient wrt its parameters

policy objective and stochastic gradient wrt its parameters

#### SAC: Objectives and Optimization

- Critic Soft Q-function
  - minimize square error
  - $\bar{\theta}$  exponential moving average of soft Q-function weights to stabilize training (DQN)

$$J_Q(\theta) = \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \mathcal{D}} \left[ \frac{1}{2} \left( Q_\theta(\mathbf{s}_t, \mathbf{a}_t) - \left( r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[ V_{\bar{\theta}}(\mathbf{s}_{t+1}) \right] \right) \right)^2 \right]$$
$$V(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t \sim \pi} \left[ Q(\mathbf{s}_t, \mathbf{a}_t) - \alpha \log \pi(\mathbf{a}_t | \mathbf{s}_t) \right]$$

 $\hat{\nabla}_{\theta} J_Q(\theta) = \nabla_{\theta} Q_{\theta}(\mathbf{a}_t, \mathbf{s}_t) \left( Q_{\theta}(\mathbf{s}_t, \mathbf{a}_t) - \left( r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \left( Q_{\bar{\theta}}(\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) - \alpha \log \left( \pi_{\phi}(\mathbf{a}_{t+1} | \mathbf{s}_{t+1}) \right) \right) \right)$ 

#### SAC: Objectives and Optimization

• Actor - Policy 
$$\pi_{\text{new}} = \arg\min_{\pi' \in \Pi} D_{\text{KL}} \left( \pi'(\cdot | \mathbf{s}_t) \left\| \frac{\exp\left(\frac{1}{\alpha} Q^{\pi_{\text{old}}}(\mathbf{s}_t, \cdot)\right)}{Z^{\pi_{\text{old}}}(\mathbf{s}_t)} \right)$$

• multiply by alpha and ignoring the normalization Z

$$J_{\pi}(\phi) = \mathbb{E}_{\mathbf{s}_{t} \sim \mathcal{D}} \left[ \mathbb{E}_{\mathbf{a}_{t} \sim \pi_{\phi}} \left[ \alpha \log \left( \pi_{\phi}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) - Q_{\theta}(\mathbf{s}_{t}, \mathbf{s}_{t}) \right] \right]$$

- reparameterize with neural network f  $\mathbf{a}_t = f_{\phi}(\epsilon_t; \mathbf{s}_t)$ 
  - epsilon: input noise vector, sampled from a fixed distribution (spherical Gaussian)

$$J_{\pi}(\phi) = \mathbb{E}_{\mathbf{s}_{t} \sim \mathcal{D}, \epsilon_{t} \sim \mathcal{N}} \left[ \alpha \log \pi_{\phi}(f_{\phi}(\epsilon_{t}; \mathbf{s}_{t}) | \mathbf{s}_{t}) - Q_{\theta}(\mathbf{s}_{t}, f_{\phi}(\epsilon_{t}; \mathbf{s}_{t})) \right]$$
$$\hat{\nabla}_{\phi} J_{\pi}(\phi) = \nabla_{\phi} \alpha \log \left( \pi_{\phi}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) + \left( \nabla_{\mathbf{a}_{t}} \alpha \log \left( \pi_{\phi}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) - \nabla_{\mathbf{a}_{t}} Q(\mathbf{s}_{t}, \mathbf{a}_{t}) \right) \nabla_{\phi} f_{\phi}(\epsilon_{t}; \mathbf{s}_{t})$$

Unbiased gradient estimator that extends DDPG stype policy gradients to any tractable stochastic policy

## SAC: Algorithm



Note

- Original paper learns V to stabilize training
- But in the second paper, V is not learned (reasons unclear)

#### **Experimental Results**

- Tasks
  - A range of continuous control tasks from the OpenAI gym benchmark suite
  - RL-Lab implementation of the Humanoid task
  - The easier tasks can be solved by a wide range of different algorithms, the more complex benchmarks, such as the 21-dimensional Humanoid (rllab) are exceptionally difficult to solve with off-policy algorithms.
- Baselines:
  - DDPG, SQL, PPO, TD3 (concurrent)
  - TD3 is an extension to DDPG that first applied the double Q-learning trick to continuous control along with other improvements.

#### SAC: Results



#### **Experimental Results: Ablation Study**

- How does the stochasticity of the policy and entropy maximization affect the performance?
- Comparison with a deterministic variant of SAC that does not maximize the entropy and that closely resembles DDPG



#### Experimental Results: Hyperparameter Sensitivity



https://arxiv.org/abs/1801.01290

#### Limitation

- Unfortunately, SAC also suffers from brittleness to the alpha temperature hyperparameter that controls exploration
  - -> automatic temperature tuning!

## Soft Actor-Critic Algorithms and Applications

Thomas Haarnoja, Aurick Zhou, Kristian Hartikainen, George Tucker, Sehoon Ha, Jie Tan, Tikash Kumar, Henry Zhu, Abhishek Gupta, Pieter Abbeel, Sergey Levine

#### Contributions

• Adaptive temperature coefficient

$$\sum_{t} \mathbb{E}_{(\mathbf{s}_{t},\mathbf{a}_{t})\sim\rho_{\pi}} \left[ r(\mathbf{s}_{t},\mathbf{a}_{t}) + \alpha \mathcal{H}(\pi(\cdot|\mathbf{s}_{t})) \right]$$

• Extend to real-world tasks such as locomotion for a quadrupedal robot and robotic manipulation with a dexterous hand





- Dexterous Hand Manipulations
- 20 hour end-to-end learning
- valve position as input: SAC 3 hours vs. PPO 7.4 hours



#### Automatic Temperature Tuning

- Choosing the optimal temperature is non-trivial (tuned for each task)
- Constrained optimization problem:

$$\max \sum_{t} \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \rho_{\pi}} \left[ r(\mathbf{s}_t, \mathbf{a}_t) + \alpha \mathcal{H}(\pi(\cdot | \mathbf{s}_t)) \right]$$

$$\max_{\pi_{0:T}} \mathbb{E}_{\rho_{\pi}} \left[ \sum_{t=0}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right] \text{ s.t. } \mathbb{E}_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim \rho_{\pi}} \left[ -\log(\pi_{t}(\mathbf{a}_{t} | \mathbf{s}_{t})) \right] \geq \mathcal{H} \ \forall t$$

#### **Dual Problem for the Constrained Optimization**

Unroll the expectation

$$\max_{\pi_0} \left( \mathbb{E}\left[ r(\mathbf{s}_0, \mathbf{a}_0) \right] + \max_{\pi_1} \left( \mathbb{E}\left[ \dots \right] + \max_{\pi_T} \mathbb{E}\left[ r(\mathbf{s}_T, \mathbf{a}_T) \right] \right) \right)$$

For the last time step in the trajectory

$$\max_{\pi_T} \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \rho_{\pi}} \left[ r(\mathbf{s}_T, \mathbf{a}_T) \right] = \min_{\alpha_T \ge 0} \max_{\pi_T} \mathbb{E} \left[ r(\mathbf{s}_T, \mathbf{a}_T) - \alpha_T \log \pi(\mathbf{a}_T | \mathbf{s}_T) \right] - \alpha_T \mathcal{H}_{\mathbf{s}_T}$$

$$\arg\min_{\alpha_T} \mathbb{E}_{\mathbf{s}_t, \mathbf{a}_t \sim \pi_t^*} \left[ -\alpha_T \log \pi_T^*(\mathbf{a}_T | \mathbf{s}_T; \alpha_T) - \alpha_T \mathcal{H} \right].$$

#### **Dual Problem for the Constrained Optimization**

Similarly, for the previous time step

$$\max_{\pi_{T-1}} \left( \mathbb{E} \left[ r(\mathbf{s}_{T-1}, \mathbf{a}_{T-1}) \right] + \max_{\pi_T} \mathbb{E} \left[ r(\mathbf{s}_T, \mathbf{a}_T) \right] \right)$$
(16)  
$$= \max_{\pi_{T-1}} \left( Q_{T-1}^*(\mathbf{s}_{T-1}, \mathbf{a}_{T-1}) - \alpha_T \mathcal{H} \right)$$
$$= \min_{\alpha_{T-1} \ge 0} \max_{\pi_{T-1}} \left( \mathbb{E} \left[ Q_{T-1}^*(\mathbf{s}_{T-1}, \mathbf{a}_{T-1}) \right] - \mathbb{E} \left[ \alpha_{T-1} \log \pi(\mathbf{a}_{T-1} | \mathbf{s}_{T-1}) \right] - \alpha_{T-1} \mathcal{H} \right) + \alpha_T^* \mathcal{H}.$$

$$\alpha_t^* = \arg\min_{\alpha_t} \mathbb{E}_{\mathbf{a}_t \sim \pi_t^*} \left[ -\alpha_t \log \pi_t^*(\mathbf{a}_t | \mathbf{s}_t; \alpha_t) - \alpha_t \bar{\mathcal{H}} \right]$$

#### Algorithm 1 Soft Actor-Critic

for each iteration do

for each environment step do

$$\mathbf{a}_{t} \sim \pi_{\phi}(\mathbf{a}_{t}|\mathbf{s}_{t}) \\ \mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_{t}, \mathbf{a}_{t}) \\ \mathcal{D} \leftarrow \mathcal{D} \cup \{(\mathbf{s}_{t}, \mathbf{a}_{t}, r(\mathbf{s}_{t}, \mathbf{a}_{t}), \mathbf{s}_{t+1})\}$$

#### end for

for each gradient step do

$$\begin{array}{ll} \theta_{i} \leftarrow \theta_{i} - \lambda_{Q} \hat{\nabla}_{\theta_{i}} J_{Q}(\theta_{i}) \text{ for } i \in \{1, 2\} \\ \phi \leftarrow \phi - \lambda_{\pi} \hat{\nabla}_{\phi} J_{\pi}(\phi) \\ \alpha \leftarrow \alpha - \lambda \hat{\nabla}_{\alpha} J(\alpha) \\ \bar{\theta}_{i} \leftarrow \tau \theta_{i} + (1 - \tau) \bar{\theta}_{i} \text{ for } i \in \{1, 2\} \\ \text{end for} \\ \text{exponential moving average} \\ \text{end for} \end{array}$$

**Output:**  $\theta_1, \theta_2, \phi$ 

Initial parameters
 Initialize target network weights
 Initialize an empty replay pool

Sample action from the policy
 Sample transition from the environment
 Store the transition in the replay pool

Update the Q-function parameters
 Update policy weights
 Adjust temperature
 Update target network weights

Optimized parameters

#### **Experimental Results: RL Lab**



Figure 1: Training curves on continuous control benchmarks. Soft actor-critic (blue and yellow) performs consistently across all tasks and outperforming both on-policy and off-policy methods in the most challenging tasks.

#### Experimental Results: Robustness



• Lack of experiments on hard-exploration problem

- Lack of experiments on hard-exploration problem
- Approximating a multi-modal Boltzmann distribution with a unimodal Gaussian



- Lack of experiments on hard-exploration problem
- Approximating a multi-modal Boltzmann distribution with a unimodal Gaussian
- High-variance using automatic temperature tuning

- Lack of experiments on hard-exploration problem
- Approximating a multi-modal Boltzmann distribution with a unimodal Gaussian
- High-variance using automatic temperature tuning



#### Recap: SAC

- An off-policy maximum entropy deep reinforcement learning algorithm
  - Sample-efficient
  - Scale to high-dimensional observation/action space
  - Robustness to random seed, noise and etc.
- Theoretical Results
  - Convergence of soft policy iteration
  - Derivation of soft-actor critic algorithm
- Empirical Results
  - SAC outperforms SOTA model-free deep RL methods, including DDPG, PPO and Soft Q-learning, in terms of the policy's optimality, sample complexity and robustness.

#### Questions to test your understanding

- What is the objective in maximum entropy reinforcement learning?
- Why are off-policy methods more sample-efficient compared to on-policy methods?
- Why do we want the policy to be close to the exponential transformation of Q-value?
- What is soft policy iteration?

# Any Questions? Thank you!

#### **Background: Q-Learning**

- Q-Learning: use any behavioral policy to estimate the optimal Q\* function that maximizes the future reward
  - Directly approximate Q\* with Bellman Optimality Equation
  - Independent of policy being followed



## Max Entropy RL

- Entropy  $H(P) = \mathop{\mathrm{E}}_{x \sim P} \left[ -\log P(x) \right]$
- Entropy-regularized Reinforcement Learning

$$\pi^* = \arg \max_{\pi} \mathop{\mathrm{E}}_{\tau \sim \pi} \left[ \sum_{t=0}^{\infty} \gamma^t \left( R(s_t, a_t, s_{t+1}) + \alpha H\left(\pi(\cdot | s_t)\right) \right) \right]$$

• State Value Function & Value Function Q

$$V^{\pi}(s) = \mathop{\mathrm{E}}_{\tau \sim \pi} \left[ \sum_{t=0}^{\infty} \gamma^{t} \left( R(s_{t}, a_{t}, s_{t+1}) + \alpha H\left(\pi(\cdot|s_{t})\right) \right) \middle| s_{0} = s \right] \quad V^{\pi}(s) = \mathop{\mathrm{E}}_{a \sim \pi} \left[ Q^{\pi}(s, a) \right] + \alpha H(\pi(\cdot|s_{t}))$$
$$Q^{\pi}(s, a) = \mathop{\mathrm{E}}_{\tau \sim \pi} \left[ \sum_{t=0}^{\infty} \gamma^{t} R(s_{t}, a_{t}, s_{t+1}) + \alpha \sum_{t=1}^{\infty} \gamma^{t} H\left(\pi(\cdot|s_{t})\right) \middle| s_{0} = s, a_{0} = a \right]$$

https://spinningup.openai.com/en/latest/algorithms/sac.html

- Need the ability to generalize to unseen environment and robustness against noisy real-world environment
- Robots get damaged in the physical world
  - requires sample-efficient learning
- Examples
  - Quadrupedal Locomotion in the Real World (2 hours of training)
  - Dexterous Hand Manipulations (20 hours end-to-end learning)

• Minitaur robot (Kenneally et al., 2016)





"first example of a DRL algorithm learning underactuated quadrupedal locomotion directly in the real world without any simulation or pretraining"

- Dexterous Hand Manipulations
- 20 hour end-to-end learning
- valve position as input: SAC 3 hours vs. PPO 7.4 hours



- Sample inefficient algorithms can be problematic when deployed in the real world
  - damage to robots/humans



#### Main Problems

widespread adoption of model-free DRL is hampered by:

- expensive in terms of sample complexity
  - simple tasks require millions of steps of data collection
  - high-dimensional observations/action space require substantially more
- brittle with respect to hyperparameters
  - learning rates, exploration constants
  - set carefully to achieve good results

## Soft actor-critic

#### 1. Q-function update

Update Q-function to evaluate current policy:

$$Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \mathbb{E}_{\mathbf{s}' \sim p_{\mathbf{s}}, \mathbf{a}' \sim \pi} \left[ Q(\mathbf{s}', \mathbf{a}') - \log \pi(\mathbf{a}' | \mathbf{s}') \right]$$

This converges to  $Q^{\pi}$ .

2. Update policy Update the policy with gradient of information projection:

$$\pi_{\text{new}} = \arg\min_{\pi'} \mathcal{D}_{\text{KL}} \left( \pi'(\,\cdot\,|\mathbf{s}) \, \middle\| \, \frac{1}{Z} \exp Q^{\pi_{\text{old}}}(\mathbf{s},\,\cdot\,) \right)$$

In practice, only take one gradient step on this objective

#### 3. Interact with the world, collect more data

Haarnoja, Zhou, Abbeel, L., Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning. 2018